Modeling weakly coupled homogeneous multicore fibers within an SDM simulation environment

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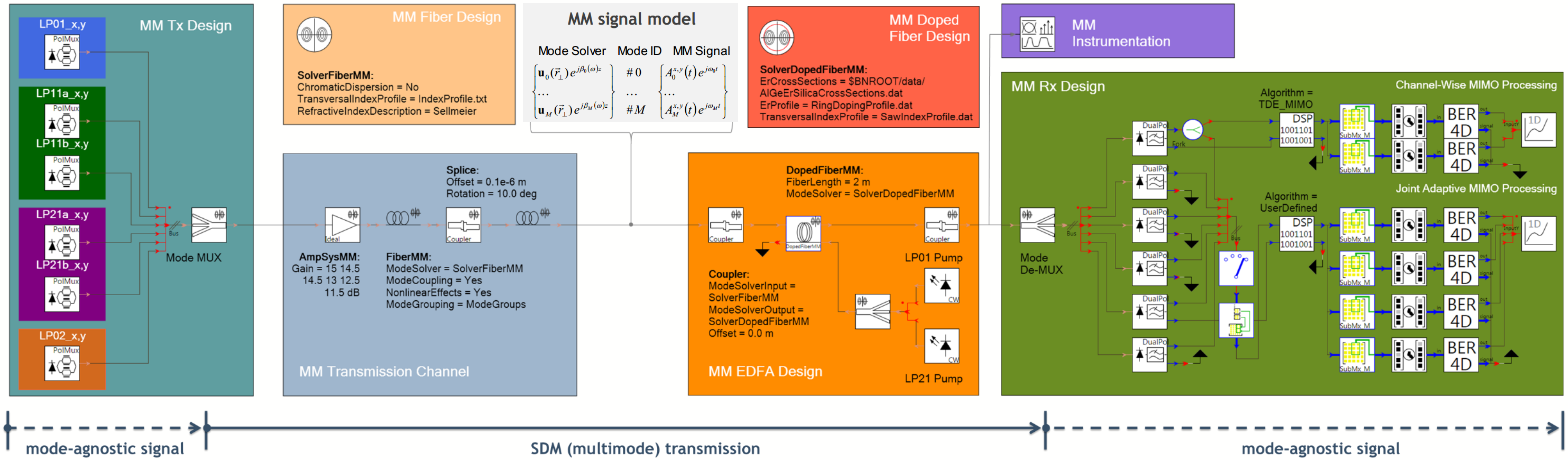
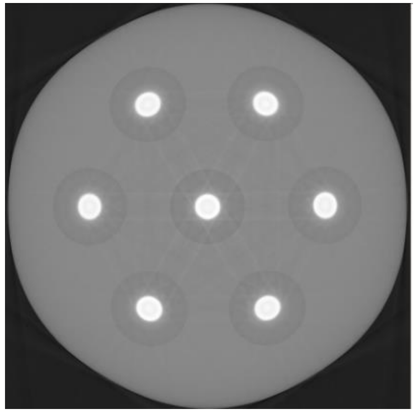
SDM Simulation Environment



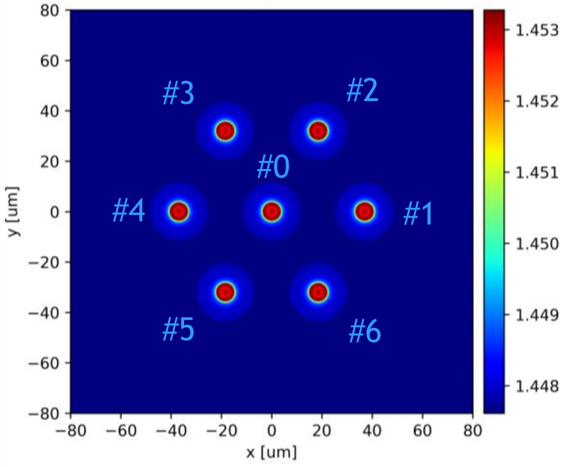
Goal of this work: identify a model for the inter-core coupling in MCFs and combine it with an existing multimode (single-core) fiber model

MCF Design

micro-photo



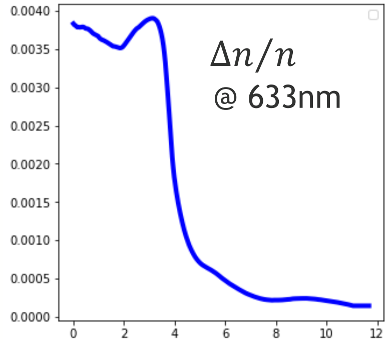
simulated RIP



- Fiber design & fabrication by Heraeus
- Operating wavelength λ_0 = 1260 nm
- Cladding / cylinder: SiO_2
- Core Rods (CRs): ~ 3.5 mol % Ge doping
- Pitch Λ ~ 37 um, Outer Diameter OD ~ 125 um
- Simulations based on the RIP of the fiber preform

RIP extraction for 1260 nm

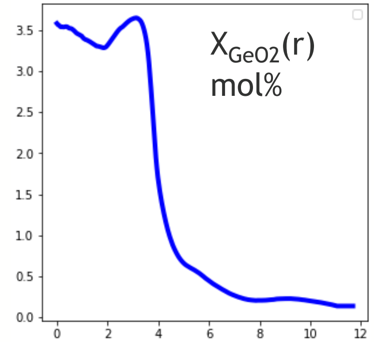
1) index contrast measured at λ_m = 633 nm



2) eq. index of GeO_2 using published data for Sellmeier coefficients (reference glass)

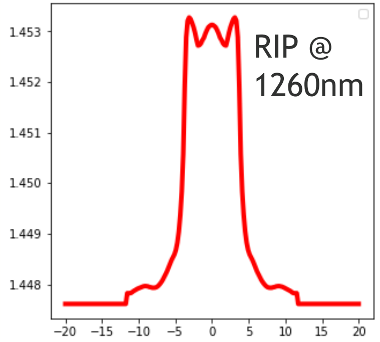
$$n^2_{GeO_2}(\lambda_m) = n^2_{SiO_2}(\lambda_m) + \frac{n^2_{ref}(\lambda_m) - n^2_{SiO_2}(\lambda_m)}{X_{ref}}$$

3) extract radial doping profile



$$X_{GeO2}(r) = \frac{n^2_{CR}(r,\lambda_m) - n^2_{SiO2}(\lambda_m)}{n^2_{GeO2}(\lambda_m) - n^2_{SiO2}(\lambda_m)}$$

4) RIP at any λ:



$$n^2_{CR}(r,\lambda) = n^2_{SiO_2}(\lambda) + X_{GeO2}(r)\left[n^2_{GeO_2}(\lambda) - n^2_{SiO_2}(\lambda)\right]$$

Numerical Method

Simulation environment: *VPIdeviceDesigner* (Python library + Jupyter)

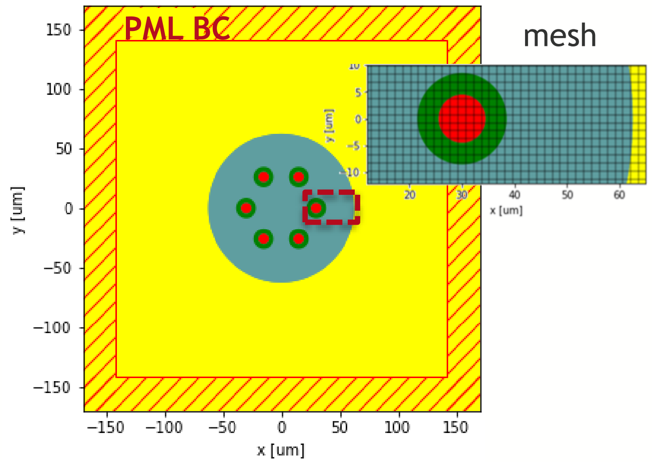
<ins>Specification of the fiber cross-section</ins>

- Straight and bent fibers (R ~ 6-140 mm)
- Step-like & gradual RIP
- Homogeneous & Heterogeneous MCFs
- Configurable trench shape
- Activation/inactivation of individual cores

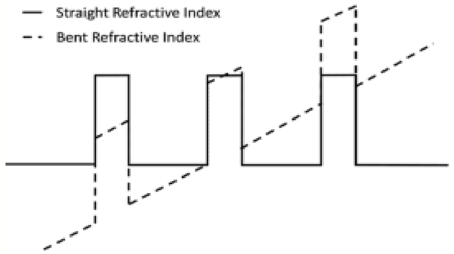
<ins>Numerical Method & Parameters</ins>

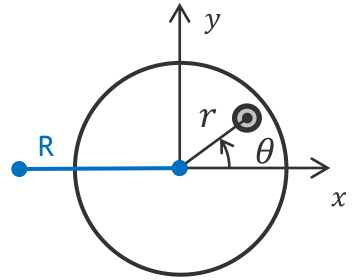
- FDM Solvers
  - Fast & accurate vectorial solver for straight fibers [1]
  - Vectorial solver in cylindrical coordinates [2] for bent fibers
  - A scalar solver for straight fibers
- Computation box of 160 um x 160 um, step size of 0.1 um, PML depth of 10 um, selected based on the analysis of numerical errors for test cases with known analytical solutions

[1] P. Lusse, et al., J. Lightwave Technol. 12 (3), 487-494 (1994)
[2] J. Xiao, H. Ni, and X. Sun, Opt. Lett. 33 (16), 1848-1850 (2008)
[3] D. Marcuse, Applied Optics, 21 (23), pp. 4208-4213 (1982).



<ins>Equivalent RIP</ins> of bent fibers for solvers that only support straight fibers [3]:





$$n_{eq}^2(r,\theta) = n^2(r,\theta)\cdot\left(1+\frac{2r}{R}\cos(\theta)\right)$$

Fiber Characteristics

- **Mode Solving** → effective index $n_{eff}(\lambda)$ and modal fields $\boldsymbol{E}(\lambda; x, y)$, $\boldsymbol{H}(\lambda; x, y)$ for $\lambda \to \lambda_i \in [\lambda_0 - \Delta\lambda, \lambda_0 + \Delta\lambda]$

- **Loss coefficient:** $\alpha = \frac{4\pi}{\lambda} Im\left(n_{eff}(\lambda)\right)$

- **Group index, Dispersion, Slope:** (i) polynomial / Padé fitting

$$n_{eff}(\lambda) = \sum^{M} a_m \lambda^m \qquad n_{eff}(\lambda) = \frac{\sum^{M} a_m \lambda^m}{\sum^{N} 1 + b_n \lambda^n}$$

(ii) followed by *analytical* differentiation

$$n_{gr}(\lambda) = n_{eff}(\lambda) - \lambda \frac{dn_{eff}(\lambda)}{d\lambda} \qquad D(\lambda) = \frac{1}{c} \frac{dn_{gr}(\lambda)}{d\lambda} \qquad S(\lambda) = \frac{dD(\lambda)}{d\lambda}$$

- **Effective Area:**

$$A_{eff}(\lambda) = \frac{\left[\iint S_z(\lambda; x, y) dxdy\right]^2}{\iint S_z^2(\lambda; x, y) dxdy}$$

with the Poynting vector:

$$\boldsymbol{S}(\lambda; x, y) = \frac{1}{2} Re[\boldsymbol{E}(\lambda; x, y) \times \boldsymbol{H}^*(\lambda; x, y)]$$

Fiber Characteristics

- **Far Field: 2D-FFT of the modal fields**

$$\tilde{I}(f_x, f_y) = f_z^2 \left| \iint dxdy\, E(x,y)\, e^{[2\pi j(f_x x + f_y y)]} \right|^2$$

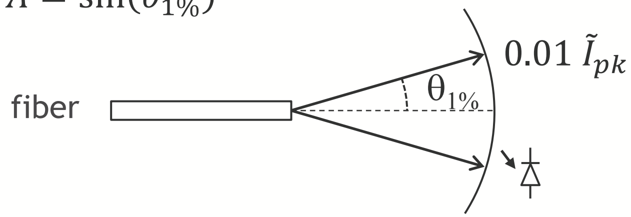
$$f_x = \frac{sin\theta_x}{\lambda}, f_y = \frac{sin\theta_y}{\lambda}, f_z = \sqrt{1 - f_x^2 - f_y^2}$$

- **Mode Field Diameter: ITU-T G.650.1**

$$MFD = \frac{\lambda}{\pi} \sqrt{\frac{2 \int \tilde{I}(\theta) \sin\theta \cos\theta \, d\theta}{\int \tilde{I}(\theta) \sin^3\theta \cos\theta \, d\theta}}$$

- **Numerical Aperture: IEC 60793-1-43**

$$NA = \sin(\theta_{1\%})$$



$$NA_{th} = \sqrt{n_{core}^2 - n_{clad}^2}$$

- **Intercore coupling coefficients**

refractive index of MCF (all cores)

refractive index of core j

$$\kappa_{ij} = \frac{2\pi c \epsilon_0}{\lambda_0} \frac{\iint dxdy\left(n^2 - n_j^2\right) \boldsymbol{E}_i^* \cdot \boldsymbol{E}_j}{\iint \boldsymbol{z} \cdot \left[\boldsymbol{E}_i^* \times \boldsymbol{H}_i + \boldsymbol{E}_i \times \boldsymbol{H}_i^*\right] dxdy}$$

Cut-Off Wavelength

- **Theoretical cut-off wavelength λ_{th}**

  shortest wavelength for which the mode solver finds just one guided mode

- **Fiber and Cable Cut-Off according to ITU-T G650.1**

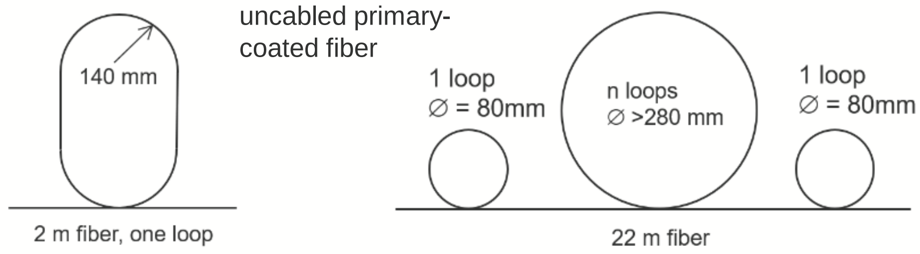
  calculated by analyzing the wavelength dependence of the bending-induced loss for the fundamental (LP_{01}) and 1st order higher-order modes ($LP_{11a,b}$):

  - input: fundamental and higher modes have equal powers
  - output: for the cut-off wavelength, the ratio of the output power of all modes to the output power of the fundamental mode equals 0.1 dB:

$$\frac{P_{LP01} + P_{LP11a} + P_{LP11b}}{P_{LP01}} = 10^{0.01}$$

Measurement Setup

Fiber Cut-Off λ_c Cable Cut-Off λ_{cc}



$$P_m(\lambda) = P_m^{(in)}(\lambda) \prod_k Exp\left[-\alpha_m^{(k)}\left(\lambda, R^{(k)}\right) L^{(k)}\right]$$

where index $m \rightarrow$ core modes, index $k \rightarrow$ straight & bent fiber segments with lengths $L^{(k)}$ and bending radii $R^{(k)}$, and the loss coefficients $\alpha_m^{(k)}\left(\lambda, R^{(k)}\right)$ are calculated by the mode solver

Calculated Characteristics

Effective Area		62.9 mm^2
Mode field diameter (far field)		8.97 mm
Dispersion		-4.97 ps/nm/km
Dispersion Slope		9.78e-2 ps/nm^2/km
Zero-dispersion wavelength		1314.2 nm
Zero-dispersion slope		8.64e-2 ps/nm^2/km
Numerical aperture (1% level in the far field)		0.138
Coupling coefficient from central to outer core		2.08e-3 1/m
Bending Loss	Bending Radius=10 mm	0.33 dB/m
	8 mm	5.0 dB/m
	6 mm	78.6 dB/m
Theoretical cut-off wavelength		1459 nm
Fiber cut-off wavelength		1396 nm
Cable cut-off wavelength		1288 nm

Coupled-Mode Theory

Coupled Mode Theory (CMT)

$$\frac{dA_m}{dz} = -j \sum_{n \neq m} \tilde{\kappa}_{mn} A_n \exp\left[j \int \underbrace{\left(\tilde{\beta}_m(z) - \tilde{\beta}_n(z) \right)}_{\Delta\beta_{mn}(z)} dz \right]$$

random index perturbations, bending, and twist:

$$\tilde{\beta}_m(z) = \beta_m \cdot \left(1 + \varepsilon_m(z) + \frac{r_m}{R} \cos(\theta_m(z)) \right)$$

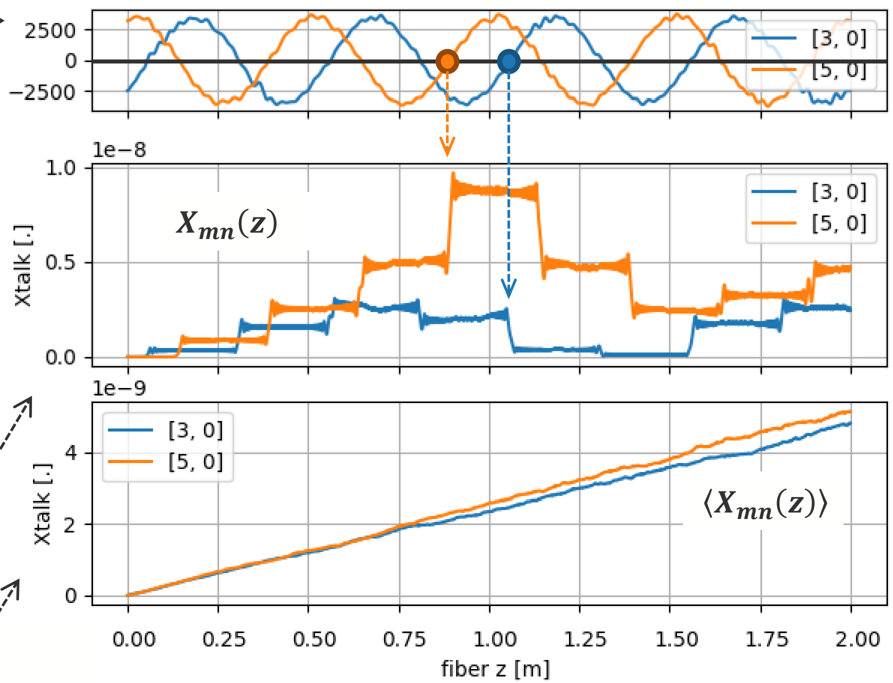
Solution

$$A_m(z) = \sum_{n \neq m} T_{mn}(z) A_n(0)$$

$X_{mn}(z) = |T_{mn}(z)|^2$ power X-Talk

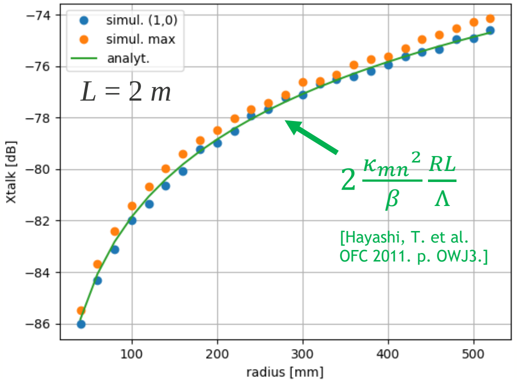
$\langle X_{mn}(z) \rangle$ averaging over 400 fiber samples

Phase Matching Points

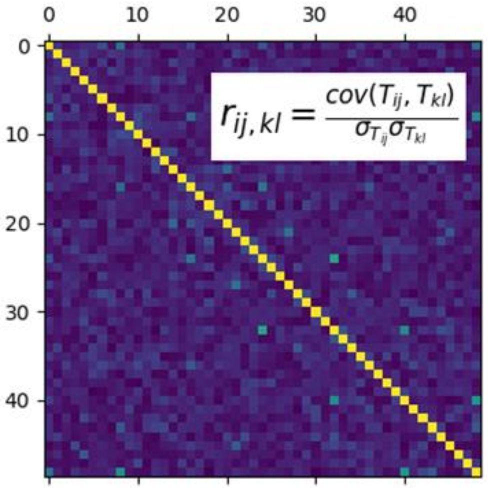


- most significant x-talk contributions occur at phase matching points (PMPs)
- x-talk fields from different cores can be summed in powers

X-talk vs. bending R



Pearson correlation for T_{mn}



PMP Model for Inter-Core Coupling

The result of the CMT study:

the intercore coupling in homogeneous WC-MCFs can be well approximated by the sum of discrete contributions from the PMPs.

The coupling strength at each PMP has deterministic magnitude and random phase.

The PMP model parameters:

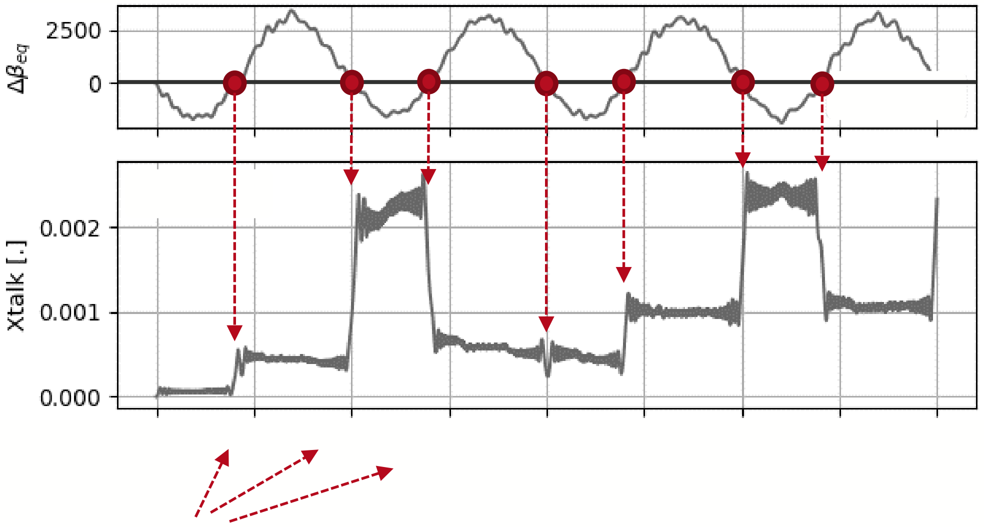
R bending radius [m]

Λ distance between cores *m* and *n* [m]

γ twist rate [rad/m]

β propagation constant [1/m]

κ coupling coefficient [1/m]

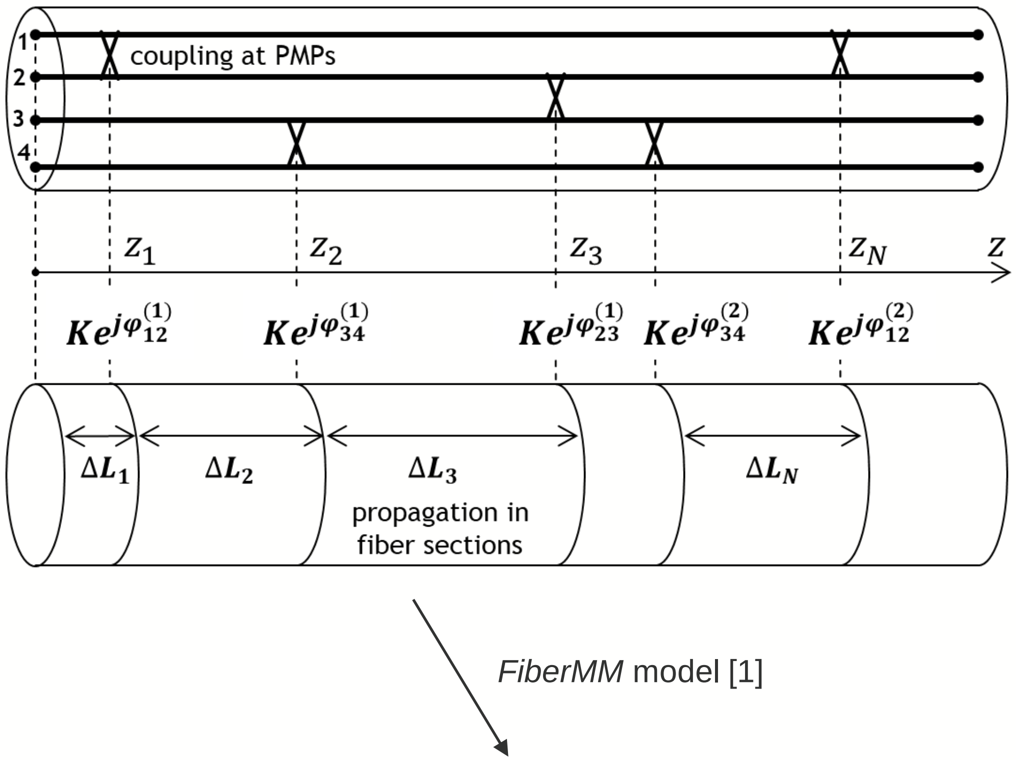


$$\boldsymbol{\Delta A_m^{(l)} = K e^{j\varphi_{mn}^{(l)}} A_n}$$

$\varphi_{mn}^{(l)}$ → uniform distribution 0..2π

$$K = \kappa\sqrt{2\pi R/\beta\Lambda\gamma} \qquad N_{PMP} = L\gamma/\pi$$

Propagation Fiber Model - Concept

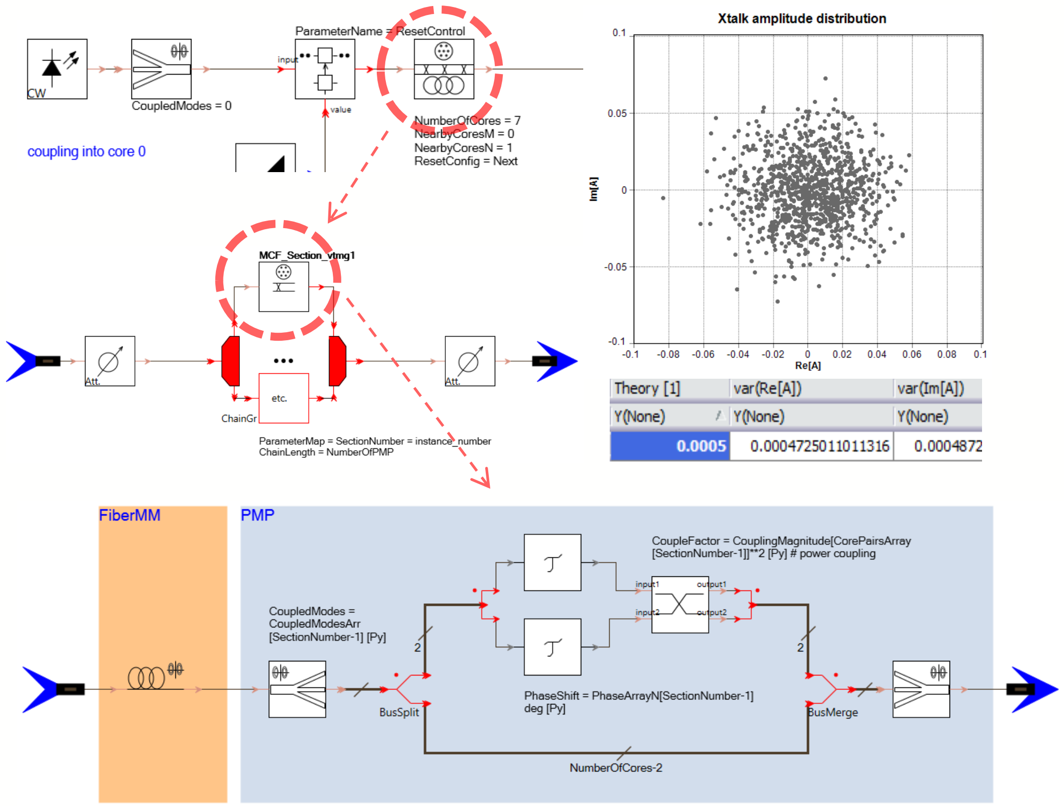


$$\frac{\partial \mathbf{A}_a}{\partial z} = \left[-\boldsymbol{\beta}'_a \frac{\partial}{\partial t} + j\boldsymbol{\beta}''_a \frac{\partial}{\partial t^2} - \frac{\boldsymbol{\alpha}_a}{2}\right]\mathbf{A}_a + \left[-j\gamma\left(\kappa_{aa}|\mathbf{A}_a|^2 + \sum_{w\neq a}\kappa_{aw}|\mathbf{A}_w|^2\right)\right]\boldsymbol{A}_a$$

- Generate N random uniformly distributed PMPs.
- Assign randomly chosen pairs of the nearby cores to each PMP.
- Couple signals at PMPs with deterministic magnitudes and random phases.
- Apply multimode fiber model [1] between PMPs to simulate the skew, PMD, CD, nonlinearities, and intra-core mode coupling.

[1] I. Koltchanov, S. Dris, A. Uvarov, and A. Richter, "Requirements for simulation-aided design of SDM systems," in OFC 2018, paper M4B.6.

Propagation Fiber Model - Prototype



Implementation in the *VPIdesignSuite*™ simulation environment

- A chain of *FiberMM* models simulates signal propagation in fiber sections between PMPs.
- The *X_Coupler* model simulates the inter-core coupling.
- Delay elements *DelaySignal* add random phase shifts.
- A Python script generates random PMP locations and assigns randomly chosen fiber cores to each PMP.

Statistical crosstalk characteristics that have been calculated using the prototype are in good agreement with the analytical predictions.

Next steps

- Apply the new model for simulations of the SDM system demonstrator in the SAMOA-NET project.
- Numerical studies for fiber design optimization: the number, position, and the size of the fiber cores.

Summary

- Implemented an MCF model for calculation of the fiber characteristics based on fiber design parameters.
- Developed a concept and prototype for the propagation model. In addition to the inter-core coupling, it can simulate CD, PMD, nonlinearity, and inter-core skew.
- The new model is integrated into our SDM environment to simulate entire transmission systems.

GEFÖRDERT VOM
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